Sanjay Ghodawat University, Kolhapur

2020-21

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EXM/P/09/00

Year and Program: SY M.Sc. | **School of Science** | **Department: Mathematics**

Course Code: MTS607 | **Course Title: Integral Equation** | **Semester :III**

Day : Friday | **End Semester Examination** | **Time:2 Hrs.**

Date: 18.12.2020 | **Max Marks:50**

Instructions: 1) All Questions are compulsory.

2)Figures to the right indicate full marks.

		Marks	Bloom's Level	CO
Q.1	**Answer the following (any Two)**			
a)	If a is constant and $n \in N$ then prove that $\underbrace{\int_a^x \int_a^x \cdots \int_a^x}_{n \text{ times}} f(t)(dt)^n = \frac{1}{(n-1)!}\int_a^x (x-t)^{n-1} f(t)\,dt.$	8	L2	CO1
b)	Reduce the integral equation $y(x) = \lambda \int_0^l k(x,t)y(t)\,dt$ where $K(x,t) = \begin{cases} \frac{t(l-x)}{l} & ;0 < t < x \\ \frac{x(1-t)}{l} & ;x < t < l \end{cases}$ into an equivalent boundary value problem.	8	L3	CO1
c)	Find the eigen values and eigen functions of the homogeneous integral equation $g(s) = \lambda \int_0^{2\pi} \sin(s+t)\, g(t)\, dt$	8	L5	CO2
Q.2	**Answer the following (any two)**			
a)	Find the resolvent kernel associated with the kernel $K(s,t) = (1+s)(1-t)$ in the interval $(0,1)$	8	L3	CO2

b) Solve the integral equation by the method of successive approximation. 8 L3 CO3

$$g(s) = 1 - \int_0^s (s-t)\, g(t)\, dt \; ; g_0(s) = 0$$

c) Solve the integral equation $g(x) = (x+1) + \int_0^x \left[1 + 2(x-y)\right] g(y)\, dy$ 8 L3 CO3

by method of differentiation.

Q.3 Answer the following (any two)

a) Solve the symmetric integral equation 9 L5 CO4

$g(s) = (s+1)^2 + \int_{-1}^{1} \left(st + s^2 t^2\right) g(t)\, dt$ by using Hilbert Schmidt theorem.

b) i) If λ_1 is the smallest eigen value of the kernel K then show that 4 L1 CO4

$$\frac{1}{|\lambda_1|} = \max \left[\frac{|(K\phi, \phi)|}{\|\phi\|^2} \right]$$

ii) Find resolvent kernel of the volterra type kernel $K(s,t) = \sin(s-t)$ 5 L3 CO5

c) Solve the Abel's integral equation $\int_0^s \frac{g(s)}{(s-t)^{1/2}} dt = 1 + s + s^2$ by using 9 L5 CO5

Laplace transform.

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